Closing Thurs: HW 9.4

Closing Tues: HW 9.5

HW Help: MSC 12:30-4:30 Mon-Thu

Entry Task: Differentiate

$$A)\ y = 5x^3 - \frac{x^{10}}{2}$$

$$B)\ y = x\sqrt{x} - 6 + \frac{3}{\sqrt{x}}$$

$$C)\ y = 5(2x)^3$$

$$D)\ y = 2\left(\frac{x^2}{3}\right)\left(\frac{9}{x^4}\right)x^3$$

Recall: *Power, Sum, Coeff. Rules*

$$\frac{d}{dx}(x^n) = n\,x^{n-1}.$$

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x).$$

$$\frac{d}{dx}\left(cf(x)\right) = cf'(x).$$

Steps:
1. Expand.
2. Rewrite Powers.
3. Use power rule.

9.5: Marginal Analysis and the Product/Quotient Rules

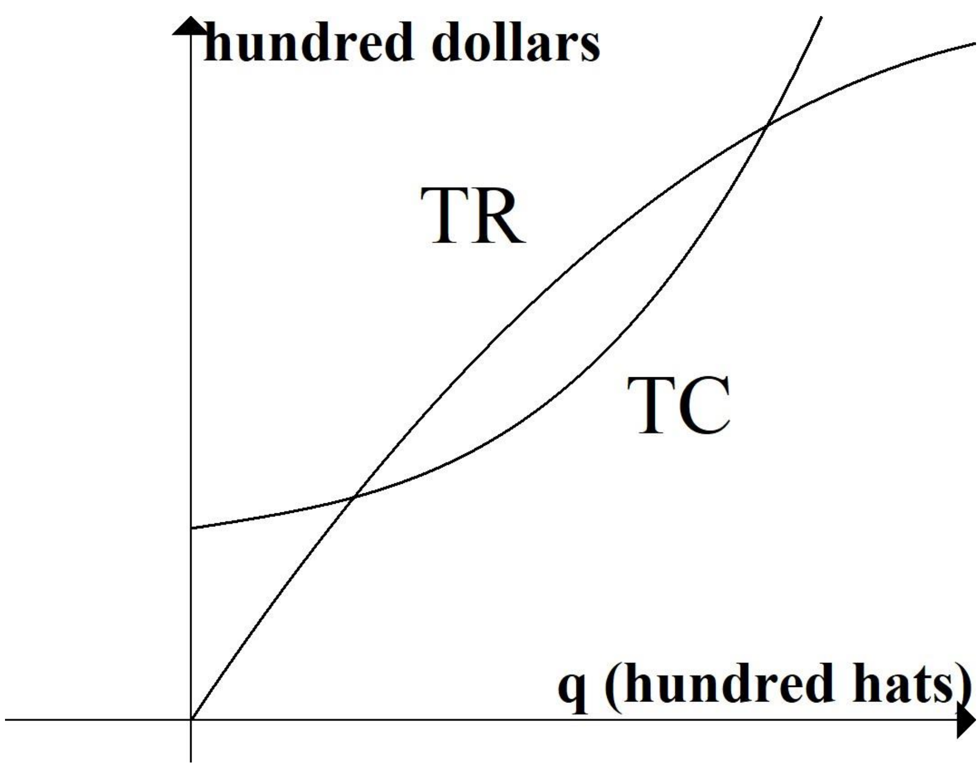
Example: Your company produces and sells hats. Based on data from recent months, you estimate:

Total Revenue:

$$TR(q) = -3.75q^2 + 28.5q$$

Total Cost:

$$TC(q) = 2q^3 - 0.4q^2 + 3q + 15$$



These functions give revenue and cost in a month where you produce

q hundred hats

$TR(q), TC(q)$ are in hundred dollars

Quick Review Question:

What is TR(0)? What is TC(0)? What do these represent?

Numerical Review:

When q = 1 hundred hats (100 hats):

$$TR(1) = -3.75(1)^2 + 28.5(1)$$
$$= 24.75 \quad \text{hundred dollars}$$
$$TC(1) = 2(1)^3 - 0.4(1)^2 + 3(1) + 15$$
$$= 19.6 \quad \text{hundred dollars}$$

$$\text{Profit} = P(1) = TR(1) - TC(1)$$
$$= 24.75 - 19.6$$
$$= 5.15 \text{ hundred dollars}$$

Thus, if you produce and sell exactly 100 hats this month, then your profit will be $515.

New Definition

In Math 112, we are going to redefine Marginal Revenue and Marginal Cost as follows:

$$MR(q) = TR'(q)$$
$$MC(q) = TC'(q)$$

Q: Using this new definition and our derivative rules, find MR(q) and MC(q).

Q: What is MR(2)?
What is MC(2)?
What are the units?

Comparing the Math 111 and 112 definitions of MR and MC

Math 111:

MR(2) = change in revenue you go from selling 200 to 201 hats
= TR(2.01) – TR(2)
= 0.134625 hundred dollars (13.46 dollars)

MC(2) = change in cost you go from making 200 to 201 hats
= TC(2.01) – TC(2)
= 0.255162 hundred dollars (25.52 dollars)

Math 112:

MR(2) = TR'(2) = 13.50 dollars/hat

MC(2) = TC'(2) = 25.40 dollars/hat

NOTES:

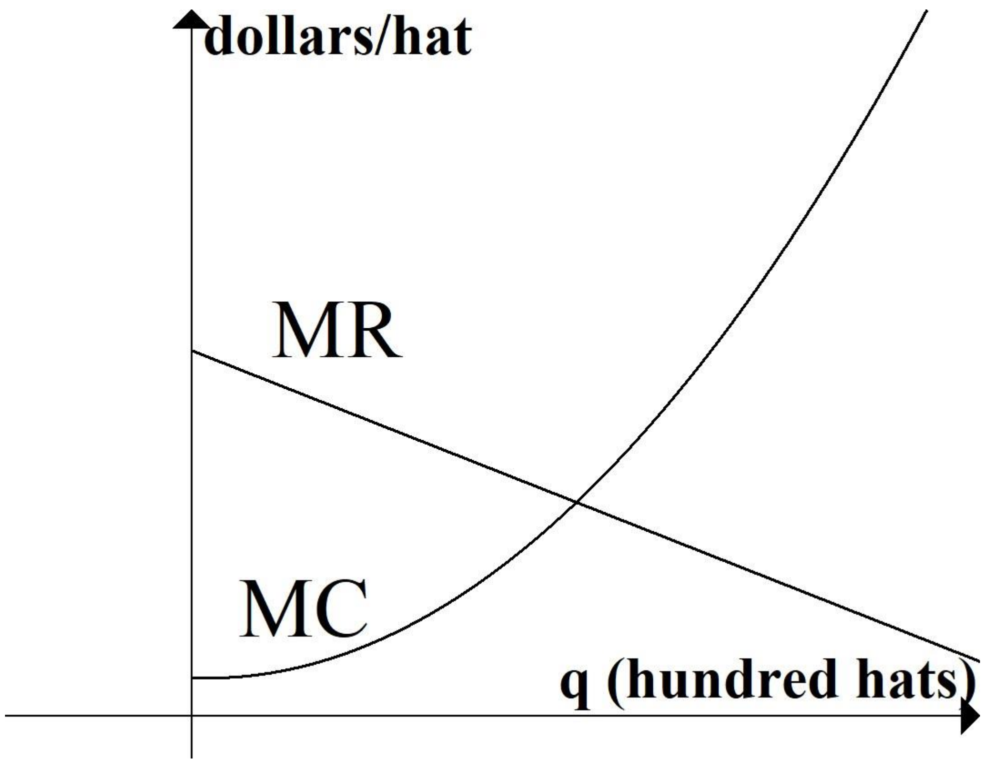
1. The Math 111 and Math 112 definitions for MR and MC differ by 5 and 12 cents, respectively. (they are *close* to the same).
2. The derivative definition is much easier to compute and use.
3. We use the Math 111 definition to interpret and analyze our results.

Marginal Revenue:

$$MR(q) = -7.5q + 28.5$$

Marginal Cost:

$$MC(q) = 6q^2 - 0.8q + 3$$



Marginal Analysis

Recall a big observation from Math 111:

If $MR(q) > MC(q)$, then profit is increasing at q.
(Revenue has a higher slope than cost)

If $MR(q) < MC(q)$, then profit is decreasing at q.
(Revenue has a lower slope than cost)

Thus, profit is maximized at the quantity when it switches from $MR(q) > MC(q)$ to $MR(q) < MC(q)$

In other words, **profit is maximized where $\boldsymbol{MR(q) = MC(q)}$.**
(match slopes)

In our example:

$$TR(q) = -3.75q^2 + 28.5q$$
$$TC(q) = 2q^3 - 0.4q^2 + 3q + 15$$

To **maximize profit**:

Step 1: Find MR(q) and MC(q):

$$MR(q) = TR'(q) = -7.5q + 28.5$$
$$MC(q) = TC'(q) = 6q^2 - 0.8q + 3$$

Step 2: Solve $MR(q) = MC(q)$

Three more derivative rules.

PRODUCT RULE: $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$.

QUOTIENT RULE:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

You try: Differentiate

1. $y = x^2(x^3 + 1)$

3. $y = (x^2 + 3x)(\sqrt{x} - 5x^3)$

2. $y = \frac{5}{x^3}$

4. $y = \frac{x^5}{3x^3 - x^5}$

CHAIN RULE:

$$\frac{d}{dx}(f(g(x))) = f'\big(g(x)\big)f'(x)$$